

Property VIII

- (i) $\sin^{-1} x + \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$ if $-1 \leq x, y \leq 1$ and $x^2 + y^2 \leq 1$ }
Or
if $xy < 0$ and $x^2 + y^2 > 1$ }
- (ii) $\sin^{-1} x - \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$ if $-1 \leq x, y \leq 1$ and $x^2 + y^2 \leq 1$ }
Or
if $xy > 0$ and $x^2 + y^2 > 1$ }

Property IX

- (i) $\cos^{-1} x + \cos^{-1} y = \cos^{-1} \{xy - \sqrt{1-x^2} \sqrt{1-y^2}\}$ if $-1 \leq x, y \leq 1$ and $x + y \geq 0$
- (ii) $\cos^{-1} x - \cos^{-1} y = \cos^{-1} \{xy + \sqrt{1-x^2} \sqrt{1-y^2}\}$ if $-1 \leq x, y \leq 1$ and $x \leq y$

Property X

- (i) $2 \sin^{-1} x = \sin^{-1} (2x\sqrt{1-x^2})$, $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$
- (ii) $3 \sin^{-1} x = \sin^{-1} (3x - 4x^3)$, $-\frac{1}{2} \leq x \leq \frac{1}{2}$

Property XI

- (i) $2 \cos^{-1} x = \cos^{-1} (2x^2 - 1)$, $0 \leq x \leq 1$
- (ii) $3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x)$, $\frac{1}{2} \leq x \leq 1$

Property XII

- (i) $2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$, $-1 < x < 1$
- (ii) $3 \tan^{-1} x = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right)$, $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$

Property XIII

- (i) $2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right)$ if $-1 \leq x \leq 1$
- (ii) $2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right)$ if $0 \leq x < \infty$