

Formula

Inverse Trigonometric Functions

Function	Domain	Range (Principal value)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \tan^{-1} x$	\mathbb{R}	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
$y = \operatorname{cosec}^{-1} x$	$\mathbb{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
$y = \sec^{-1} x$	$\mathbb{R} - [-1, 1]$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$
$y = \cot^{-1} x$	\mathbb{R}	$(0, \pi)$

Property I

- (i) $\sin^{-1}(\sin \theta) = \theta, \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
- (ii) $\cos^{-1}(\cos \theta) = \theta, \quad \theta \in [0, \pi]$
- (iii) $\tan^{-1}(\tan \theta) = \theta, \quad \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
- (iv) $\operatorname{cosec}^{-1}(\operatorname{cosec} \theta) = \theta, \quad \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
- (v) $\sec^{-1}(\sec \theta) = \theta, \quad \theta \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$
- (vi) $\cot^{-1}(\cot \theta) = \theta, \quad \theta \in (0, \pi)$

Property II

- (i) $\sin(\sin^{-1} x) = x, \quad x \in [-1, 1]$
- (ii) $\cos(\cos^{-1} x) = x, \quad x \in [-1, 1]$
- (iii) $\tan(\tan^{-1} x) = x, \quad x \in \mathbb{R}$
- (iv) $\operatorname{cosec}(\operatorname{cosec}^{-1} x) = x, \quad x \in \mathbb{R} - [-1, 1]$
- (v) $\sec(\sec^{-1} x) = x, \quad x \in \mathbb{R} - [-1, 1]$
- (vi) $\cot(\cot^{-1} x) = x, \quad x \in \mathbb{R}$

Property III

- (i) $\sin^{-1} x = \cos^{-1} \sqrt{1 - x^2}$
 $= \operatorname{cosec}^{-1} \left(\frac{1}{x} \right)$
- (ii) $\cos^{-1} x = \sin^{-1} \sqrt{1 - x^2}$
 $= \sec^{-1} \left(\frac{1}{x} \right)$
- (iii) $\tan^{-1} x = \cot^{-1} \left(\frac{1}{x} \right)$

Property IV

- (i) $\sin^{-1}(-x) = -\sin^{-1} x, \quad x \in [-1, 1]$
- (ii) $\cos^{-1}(-x) = \pi - \cos^{-1} x, \quad x \in [-1, 1]$
- (iii) $\tan^{-1}(-x) = -\tan^{-1} x, \quad x \in R$
- (iv) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x, \quad |x| \geq 1$
- (v) $\sec^{-1}(-x) = \pi - \sec^{-1} x, \quad |x| \geq 1$
- (vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x, \quad x \in R$

Property V

- (i) $\sin^{-1} \left(\frac{1}{x} \right) = \operatorname{cosec}^{-1} x, \quad x \in (-\infty, -1] \cup [1, \infty)$
- (ii) $\cos^{-1} \left(\frac{1}{x} \right) = \sec^{-1} x, \quad x \in (-\infty, -1] \cup [1, \infty)$
- (iii) $\tan^{-1} \left(\frac{1}{x} \right) = \cot^{-1} x, \quad x > 0$

Property VI

- (i) $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \quad x \in [-1, 1]$
- (ii) $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}, \quad x \in R$
- (iii) $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}, \quad |x| \geq 1$

Property VII

- (i) $\tan^{-1} x + \tan^{-1} y \begin{cases} = \tan^{-1} \left(\frac{x+y}{1-xy} \right), & xy < 1 \\ = \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right), & x > 0, y > 0 \text{ and } xy > 1 \end{cases}$
- (ii) $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \left(\frac{x-y}{1+xy} \right), \quad xy > -1$

Property VIII

$$(i) \quad \sin^{-1} x + \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} + y\sqrt{1-x^2}\} \quad \text{if } -1 \leq x, y \leq 1 \text{ and } x^2 + y^2 \leq 1$$

Or

$$\text{if } xy < 0 \text{ and } x^2 + y^2 > 1$$

$$(ii) \quad \sin^{-1} x - \sin^{-1} y = \sin^{-1} \{x\sqrt{1-y^2} - y\sqrt{1-x^2}\} \quad \text{if } -1 \leq x, y \leq 1 \text{ and } x^2 + y^2 \leq 1$$

Or

$$\text{if } xy > 0 \text{ and } x^2 + y^2 > 1$$

Property IX

$$(i) \quad \cos^{-1} x + \cos^{-1} y = \cos^{-1} \{xy - \sqrt{1-x^2} \sqrt{1-y^2}\} \quad \text{if } -1 \leq x, y \leq 1 \text{ and } x + y \geq 0$$

$$(ii) \quad \cos^{-1} x - \cos^{-1} y = \cos^{-1} \{xy + \sqrt{1-x^2} \sqrt{1-y^2}\} \quad \text{if } -1 \leq x, y \leq 1 \text{ and } x \leq y$$

Property X

$$(i) \quad 2 \sin^{-1} x = \sin^{-1} (2x\sqrt{1-x^2}), \quad -\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$$

$$(ii) \quad 3 \sin^{-1} x = \sin^{-1} (3x - 4x^3), \quad -\frac{1}{2} \leq x \leq \frac{1}{2}$$

Property XI

$$(i) \quad 2 \cos^{-1} x = \cos^{-1} (2x^2 - 1), \quad 0 \leq x \leq 1$$

$$(ii) \quad 3 \cos^{-1} x = \cos^{-1} (4x^3 - 3x), \quad \frac{1}{2} \leq x \leq 1$$

Property XII

$$(i) \quad 2 \tan^{-1} x = \tan^{-1} \left(\frac{2x}{1-x^2} \right), \quad -1 < x < 1$$

$$(ii) \quad 3 \tan^{-1} x = \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right), \quad -\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$$

Property XIII

$$(i) \quad 2 \tan^{-1} x = \sin^{-1} \left(\frac{2x}{1+x^2} \right) \quad \text{if } -1 \leq x \leq 1$$

$$(ii) \quad 2 \tan^{-1} x = \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \quad \text{if } 0 \leq x < \infty$$