Differentiation

Solution of previous years ISC Class 12 board questions

2000 to 2020

1. $If\ y = \log\sqrt{\frac{1-\cos x}{1+\cos x}}, find\ \frac{dy}{dx}.$ $(isc\ 2000)$

Solution: Given $y = \log\sqrt{\frac{1-\cos x}{1+\cos x}}$

$$= \log\sqrt{tan^2\left(\frac{x}{2}\right)}$$

$$= \log\left(\tan\frac{x}{2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{\tan\left(\frac{x}{2}\right)}.\frac{1}{2}.sec^2\left(\frac{x}{2}\right)$$

$$= \frac{1}{2}.\frac{\cos\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)}.\frac{1}{cos^2\left(\frac{x}{2}\right)}$$

$$= \frac{1}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$$

$$= \frac{1}{\sin x}$$

$= \operatorname{cosec} x$ Answer.

2. $If\ y = (\cos x)^{\cos x}, find\ \frac{dy}{dx}.$ $(isc\ 2001)$

Solution : Given $y = (\cos x)^{\cos x}$

Taking log both sides

$$\log y = \cos x \log(\cos x)$$

Differentiating each term $w.r.t.x$

$$\frac{1}{y}\frac{dy}{dx} = \cos x \frac{1}{\cos x}(-\sin x) + (-\sin x)\log(\cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = -\sin x - \sin x \log(\cos x)$$

$$\Rightarrow \frac{dy}{dx} = -y \sin x \{1 + \log(\cos x)\}$$

$$\Rightarrow \frac{dy}{dx} = -(\cos x)^x \sin x \{\log e + \log(\cos x)\}$$

$\Rightarrow \frac{dy}{dx} = -(\cos x)^x \sin x \{\log(e \cos x)\}$ Answer.

3. $If\ y = e^x \log(\tan 2x), find\ \frac{dy}{dx}.$ $(isc\ 2002)$

Solution : Given $y = e^x \log(\tan 2x)$

$$\Rightarrow \frac{dy}{dx} = e^x \frac{1}{\tan 2x}.2\ sec^2(2x) + e^x \log\tan 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = e^x \{2 \cot 2x \, . sec^2(2x) + \log \tan 2x\} \qquad \text{Answer.}$$

4. *If* $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$, *prove that* $\frac{dy}{dx} = \frac{2}{1+x^2}$. (isc 2003)

Solution : Given $y = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= \tan^{-1}\left(\frac{2\tan\theta}{1-tan^2\theta}\right)$ $\qquad$ *Let* $x = \tan\theta$

$= \tan^{-1}(\tan 2\theta)$ $\qquad$ $\therefore \theta = \tan^{-1} x$

$= 2\theta$

$= 2\tan^{-1} x$

$\therefore \frac{dy}{dx} = \frac{2}{1+x^2}$ $\qquad$ Proved.

5. *If* $y = e^{m\cos^{-1}x}$, *prove that* $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2 y$ (isc 2003)

Solution : Given $y = e^{m\cos^{-1}x}$

Taking log both sides

$\Rightarrow \quad \log y = m\cos^{-1}x$

Differentiating each term w.r.t. x

$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = -\frac{m}{\sqrt{1-x^2}}$

$\Rightarrow \quad \sqrt{1-x^2}\frac{dy}{dx} = -m\,y$

Squaring both sides

$\Rightarrow \quad (1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2 y^2$

Differentiating each term w.r.t. x

$\Rightarrow \quad (1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} + (-2x)\left(\frac{dy}{dx}\right)^2 = m^2\,2y\,\frac{dy}{dx}$

Dividing each term by $2\frac{dy}{dx}$

$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2 y$ $\qquad$ Proved

6. *If* $x^y y^x = 5$, *show that* $\frac{dy}{dx} = -\left(\frac{\log y + \frac{y}{x}}{\log x + \frac{x}{y}}\right)$ (isc 2004)

Solution : Given $x^y y^x = 5$

Taking log both sides

$\Rightarrow \quad \log x^y + \log y^x = \log 5$

$\Rightarrow \quad y\log x + x\log y = \log 5$

Differentiating each term w.r.t. x

$\Rightarrow \quad y\frac{1}{x} + \log x\frac{dy}{dx} + \log y + x\frac{1}{y}\frac{dy}{dx} = 0$

$\Rightarrow \quad \left(\log x + \frac{x}{y}\right)\frac{dy}{dx} = -\frac{y}{x} - \log y$

$$\Rightarrow \quad \frac{dy}{dx} = -\left(\frac{\log y + \frac{y}{x}}{\log x + \frac{x}{y}}\right)$$ Proved

7. *If* $x = a\, sin^3 t$ *and* $y = a\, cos^3 t$, *find* $\frac{dy}{dx}$. (*isc* 2004)

Solution : Given $x = a\, sin^3 t$ | $y = a\, cos^3 t$

Differentiating $w.r.t.\ t$

$$\Rightarrow \frac{dx}{dt} = a.3sin^2 t.\cos t \qquad \Rightarrow \frac{dy}{dt} = a\, 3\, cos^2 t.(-\sin t)$$

$$\Rightarrow \frac{dx}{dt} = 3\, a\, sin^2 t.\cos t \qquad \Rightarrow \frac{dy}{dt} = -3a\, cos^2 t.\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3a\, cos^2 t.\sin t}{3\, a\, sin^2 t.\cos t}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\cos t}{\sin t}$$

$$\Rightarrow \frac{dy}{dx} = -\cot t$$ Answer.

8. *If* $\sin(xy) + \cos(xy) = 1$ *and* $\tan(xy) \neq 1$, *then show that* $\frac{dy}{dx} = -\frac{y}{x}$. (*isc* 2005)

Solution : Given $\sin(xy) + \cos(xy) = 1$

Differentiating $w.r.t.\ x$

$$\Rightarrow \cos(xy)\left(y + x\frac{dy}{dx}\right) - \sin(xy)\left(y + x\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \cos(xy).x\frac{dy}{dx} - \sin(xy).x\frac{dy}{dx} = -\cos(xy).y + \sin(xy).y$$

$$\Rightarrow \{\cos(xy) - \sin(xy)\}\, x\frac{dy}{dx} = y\{\sin(xy) - \cos(xy)\}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y\{\sin(xy) - \cos(xy)\}}{x\{\cos(xy) - \sin(xy)\}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$ Proved.

9. *If* $x^p y^q = (x+y)^{p+q}$, *prove that* $\frac{dy}{dx} = \frac{y}{x}$. (*isc* 2005)

Solution : Given $x^p y^q = (x+y)^{p+q}$

Taking log both sides

$$\Rightarrow p\log x + q\log y = (p+q)\log(x+y)$$

Differentiating each term $w.r.t.\ x$

$$\Rightarrow p.\frac{1}{x} + q\frac{1}{y}.\frac{dy}{dx} = (p+q)\frac{1}{x+y}.\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{p}{x} + \frac{q}{y}.\frac{dy}{dx} = \frac{p+q}{x+y} + \frac{p+q}{x+y}.\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{q}{y} - \frac{p+q}{x+y}\right)\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\Rightarrow \left(\frac{qx+qy-py-qy}{y(x+y)}\right)\frac{dy}{dx} = \frac{px+qx-px-py}{x\,(x+y)}$$

$$\Rightarrow \frac{qx-py}{y\,(x+y)}.\frac{dy}{dx} = \frac{qx-py}{x\,(x+y)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(qx-py)\ y\ (x+y)}{x\ (x+y)\ (qx-py)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x}$$ Proved

10. $If\ y = e^{\sin x^2}, find\ \frac{dy}{dx}.$ $(isc\ 2006)$

Solution : Given $y = e^{\sin x^2}$

Taking log both sides

$$\Rightarrow \quad \log y = \sin x^2$$

$Differentiating\ each\ term\ w.r.t.\ \ x$

$$\Rightarrow \quad \frac{1}{y}.\frac{dy}{dx} = \cos x^2 . 2x$$

$$\Rightarrow \quad \frac{dy}{dx} = y. 2x\ \cos x^2$$

$$\Rightarrow \quad \frac{dy}{dx} = 2x. e^{\sin x^2}. \cos x^2$$ Answer.

11. $If\ y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}, prove\ that\ (1-x^2)\frac{dy}{dx} - xy = 1$ $(isc\ \ 2006)$

Solution : Given $y = \frac{\sin^{-1} x}{\sqrt{1-x^2}}$

$$\Rightarrow \quad \sqrt{1-x^2}. y = \sin^{-1} x$$

$$\Rightarrow \quad \sqrt{1-x^2}\frac{dy}{dx} + \frac{1}{2\sqrt{1-x^2}}(-2x)y = \frac{1}{\sqrt{1-x^2}}$$ (differentiating each term $w.r.t.\ x$)

$$\Rightarrow \quad (1-x^2)\frac{dy}{dx} - x\,y = 1$$ Proved.

12. $If\ e^{x+y} = xy, show\ that\ \frac{dy}{dx} = \frac{y\ (1-x)}{x\ (1-y)}$ $(isc\ \ 2007)$

Solution : Given $e^{x+y} = xy$

Taking log both sides

$$\Rightarrow \quad x + y = \log x + \log y$$

$Differentiating\ each\ term\ w.r.t.\ x$

$$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}.\frac{dy}{dx}$$

$$\Rightarrow \quad \left(1 - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x} - 1$$

$$\Rightarrow \quad \left(\frac{y-1}{y}\right)\frac{dy}{dx} = \frac{1-x}{x}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y\ (1-x)}{x\ (y-1)}$$ Proved

13. $If\ \ \sin y = x\ \sin(a+y), show\ that\ \frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$ $(isc\ \ 2007)$

Solution : Given $\sin y = x\ \sin(a+y)$

$$\Rightarrow \quad x = \frac{\sin y}{\sin(a+y)}$$

Differentiating both sides $w.r.t.\ y$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{\sin(a+y)\cos y - \cos(a+y)\sin y}{sin^2(a+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{\sin(a+y-y)}{sin^2(a+y)}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{\sin a}{sin^2(a+y)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$$ Proved

14. *Find* $\frac{dy}{dx}$ *if* $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ (*isc* 2008)

Solution : Given $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ *Let* $x = \tan\theta$

$= \tan^{-1}\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}$ $\therefore\ \theta = \tan^{-1}x$

$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}\left(\frac{2\,sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$

$= \frac{\theta}{2}$

$= \frac{1}{2}\tan^{-1}x$

$\therefore \frac{dy}{dx} = \frac{1}{2\,(1+x^2)}$ Answer

15. $If\ y = \sqrt{\frac{1-\cos x}{1+\cos x}}, find\ \frac{dy}{dx}$ (*isc* 2008)

Solution : Given $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

$= \sqrt{tan^2\left(\frac{x}{2}\right)}$

$= \tan\left(\frac{x}{2}\right)$

$\therefore \frac{dy}{dx} = \frac{1}{2}\,sec^2\left(\frac{x}{2}\right)$ Answer.

16. Using a suitable substitution, find the derivative of $\tan^{-1}\frac{4\sqrt{x}}{1-4x}$ (*isc* 2009)

Solution : Let $y = \tan^{-1}\frac{4\sqrt{x}}{1-4x}$ *Let* $2\sqrt{x} = \tan\theta$

$$= \tan^{-1}\frac{2.2\sqrt{x}}{1-(2\sqrt{x})^2} \qquad \therefore\ \theta = \tan^{-1}(2\sqrt{x})$$

$$= \tan^{-1}\left(\frac{2\tan\theta}{1-tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 2\theta)$$

$$= 2\,\theta$$

$$= 2\ \tan^{-1}(2\sqrt{x})$$

$$\therefore\ \frac{dy}{dx} = \frac{2}{1+(2\sqrt{x})^2}\,.2\,.\frac{1}{2\sqrt{x}}$$

$$= \frac{2}{\sqrt{x}\,(1+4x)}$$ Answer.

17. Find the derivative of $\sin x^2$ with respect to x^3. $(isc\ 2009)$

Solution : $Let\ \ y = \sin x^2 \qquad and\ \ z = x^3$

$$\Rightarrow\ \frac{dy}{dx} = \cos(x^2).2x \qquad and\ \frac{dz}{dx} = 3\,x^2$$

$$\therefore\ \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{2\,x\,\cos x^2}{3\,x^2}$$

$$= \frac{2\,\cos x^2}{3\,x}$$ Answer.

18. Using a suitable substitution, find the derivative of $\tan^{-1}\sqrt{\frac{a-x}{a+x}}$ with respect to x. $(isc\ 2010)$

Solution : $Let\ \ y = \tan^{-1}\sqrt{\frac{a-x}{a+x}} \qquad Let\ \ x = a\ \cos\theta$

$$= \tan^{-1}\sqrt{\frac{a-a\ \cos\theta}{a+a\ \cos\theta}} \qquad \therefore\ \theta = \cos^{-1}\left(\frac{x}{a}\right)$$

$$= \tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \tan^{-1}\sqrt{tan^2\left(\frac{\theta}{2}\right)}$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2}$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{x}{a}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\cdot\frac{-1}{\sqrt{1-\left(\frac{x}{a}\right)^2}}\cdot\frac{1}{a}$$

$$= -\frac{1}{2\sqrt{a^2-x^2}}$$ Answer.

19. *If* $y = x^x$, *prove that* $\frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$ (*isc* 2011)

Solution : Given $y = x^x$

Taking log both sides

$$\Rightarrow \log y = x \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x.\frac{1}{x} + \log x$$ (differentiating each term $w.r.t.\ x$)

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \log x$$

$$\Rightarrow \frac{1}{y}\frac{d^2y}{dx^2} + \frac{dy}{dx}.\left(-\frac{1}{y^2}\right)\frac{dy}{dx} = \frac{1}{x}$$ (differentiating again $w.r.t.x$)

$$\Rightarrow \frac{d^2y}{dx^2} - \frac{1}{y}\left(\frac{dy}{dx}\right)^2 - \frac{y}{x} = 0$$ Proved.

20. *If* $e^y(x+1) = 1$, *then show that* $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$. (*isc* 2012)

Solution : Given $e^y(x+1) = 1$

$$\Rightarrow e^y = \frac{1}{x+1}$$

$$\Rightarrow y = \log\left(\frac{1}{x+1}\right)$$

$$\Rightarrow y = -\log(x+1)$$

$$\therefore \frac{dy}{dx} = -\frac{1}{x+1}$$ and $$\left(\frac{dy}{dx}\right)^2 = \left(\frac{1}{x+1}\right)^2$$ ------------------- (1)

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{(-1)}{(x+1)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{1}{x+1}\right)^2$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$ [comparing with equation (1)] Proved

21. *If* $y = (\cot^{-1}x)^2$, *show that* $(1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$ (*isc* 2013)

Solution : Given $y = (\cot^{-1}x)^2$

Differentiating $w.r.t.\ x$

$$\Rightarrow \frac{dy}{dx} = 2\cot^{-1}x.\left(\frac{-1}{1+x^2}\right)$$

$$\Rightarrow (1+x^2)\frac{dy}{dx} = -2\cot^{-1}x$$

Differentiating again w.r.t. x

$\Rightarrow\ (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{2}{1+x^2}$

$\Rightarrow\ (1+x^2)^2\frac{d^2y}{dx^2} + 2x(1+x^2)\frac{dy}{dx} = 2$ Proved.

22. *If* $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ *, prove that* $(1-x^2)\frac{dy}{dx} = x + \frac{y}{x}$ *(isc 2014)*

Solution : Given $y = \frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ ----------- (1)

$\Rightarrow\ \sqrt{1-x^2}\,.y = x\sin^{-1}x$

Differentiating w.r.t. x

$\Rightarrow\ \sqrt{1-x^2}\frac{dy}{dx} + \frac{1}{2}.\frac{1}{\sqrt{1-x^2}}.(-2x)y = x.\frac{1}{\sqrt{1-x^2}} + \sin^{-1}x$

multiplying each term by $\sqrt{1-x^2}$

$\Rightarrow\ (1-x^2)\frac{dy}{dx} - xy = x + \sin^{-1}x\,.\sqrt{1-x^2}$

[putting value of $\sin^{-1}x$ from (1)]

$\Rightarrow\ (1-x^2)\frac{dy}{dx} - xy = x + \frac{y\sqrt{1-x^2}}{x}.\sqrt{1-x^2}$

$\Rightarrow\ (1-x^2)\frac{dy}{dx} - xy = x + \frac{y(1-x^2)}{x}$

$\Rightarrow\ (1-x^2)\frac{dy}{dx} - xy = x + \frac{y}{x} - yx$

$\Rightarrow\ (1-x^2)\frac{dy}{dx} = x + \frac{y}{x}$ Proved.

23. *If* $y = e^{m\cos^{-1}x}$*, prove that* $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2 y$ *(isc 2015)*

Solution : Given $y = e^{m\cos^{-1}x}$

Taking log both sides

$\log y = m\cos^{-1}x$

Differentiating w.r.t. x

$\frac{1}{y}\frac{dy}{dx} = m\left(-\frac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow\ \sqrt{1-x^2}\frac{dy}{dx} = -m\,y$

squaring both sides

$\Rightarrow\ (1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$

Differentiating again w.r.t. x

$\Rightarrow\ (1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} + (-2x)\left(\frac{dy}{dx}\right)^2 = m^2.2y\frac{dy}{dx}$

dividing each term by $2\frac{dy}{dx}$

$\Rightarrow\ (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2 y$ Proved. **tapatisclasses.in**

24. $If \quad \log y = \tan^{-1} x$, $prove\ that \quad (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$ (isc 2016)

Solution : Given $\log y = \tan^{-1} x$

Differentiating both sides $w.r.t.\ x$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$$

$$\Rightarrow \quad (1+x^2)\frac{dy}{dx} = y$$

$Differentiating\ again\ w.r.t\ x$

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{dy}{dx}$$

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$$ Proved

25. $If \ y = \cos(\sin x)$, $show\ that\ \frac{d^2y}{dx^2} + \tan x\ \frac{dy}{dx} + y\cos^2 x = 0$ (isc 2017)

Solution : Given $y = \cos(\sin x)$

Differentiating both sides $w.r.t.\ x$

$$\Rightarrow \quad \frac{dy}{dx} = -\sin(\sin x).\cos x$$

$again\ differentiating\ both\ sides\ w.r.t.\ x$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\cos(\sin x).\cos x.\cos x - \sin(\sin x).(-\sin x)$$

$$\Rightarrow \quad \frac{d^2y}{dx^2} = -\cos(\sin x).cos^2 x + \sin(\sin x).\sin x$$

$$Now\ L.H.S = \frac{d^2y}{dx^2} + \tan x\ \frac{dy}{dx} + y\ cos^2 x$$

$$= -\cos(\sin x).cos^2 x + \sin(\sin x).\sin x + \frac{\sin x}{\cos x}.\{-\sin(\sin x).\cos x\} + \cos(\sin x).cos^2 x$$

$$= -\cos(\sin x).cos^2 x + \sin(\sin x).\sin x - \sin x.\sin(\sin x) + \cos(\sin x).cos^2 x$$

$$= 0$$ Proved

26. $If \ x = \tan\left(\frac{1}{a}\log y\right)$, $prove\ that\ (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$ (isc 2018)

Solution : Given $x = \tan\left(\frac{1}{a}\log y\right)$

$$\Rightarrow \quad \tan^{-1} x = \frac{1}{a}\log y$$

$$\Rightarrow \quad \log y = a\ \tan^{-1} x$$

Differentiating both sides $w.r.t.\ x$

$$\Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = a\ \frac{1}{1+x^2}$$

$$\Rightarrow \quad (1+x^2)\frac{dy}{dx} = a\ y$$

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = a\ \frac{dy}{dx}$$ (Differentiating each term $w.r.t.x$)

$$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\ \frac{dy}{dx} = 0$$ Proved

27. *If* $y = e^{\sin^{-1}x}$ *and* $z = e^{-\cos^{-1}x}$, *prove that* $\frac{dy}{dz} = e^{\pi/2}$ (isc 2019)

Solution : $y = e^{\sin^{-1}x}$ and $z = e^{-\cos^{-1}x}$

$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}$ and $\frac{dz}{dx} = e^{-\cos^{-1}x}.\left(-\frac{1}{-\sqrt{1-x^2}}\right)$

$$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}}{e^{-\cos^{-1}x}.\frac{1}{\sqrt{1-x^2}}}$$

$$= \frac{e^{\sin^{-1}x}}{e^{-\cos^{-1}x}}$$

$$= e^{\sin^{-1}x+\cos^{-1}x}$$

$$= e^{\pi/2}$$ Proved

28. *Find* $\frac{dy}{dx}$ *if* $x^3 + y^3 = 3axy$ (isc 2020)

Solution : Given $x^3 + y^3 = 3axy$

Differentiating both sides $w.r.t.x$,

$$3x^2 + 3y^2\frac{dy}{dx} = 3a\left(x\frac{dy}{dx} + y\right)$$

$$\Rightarrow x^2 + y^2\frac{dy}{dx} = ax\frac{dy}{dx} + ay$$

$$\Rightarrow (y^2 - ax)\frac{dy}{dx} = ay - x^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{ay-x^2}{y^2-ax}$$ Answer

29. *If* $y = e^{m\sin^{-1}x}$, *prove that* $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$ (isc 2020)

Solution : Given $y = e^{m\sin^{-1}x}$

Taking log on both sides

$$\Rightarrow \log y = m\sin^{-1}x$$

Differentiating each term $w.r.t.x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{m}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = m\,y$$

$$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$$ (squaring both sides)

Differentiating again each term $w.r.t.x$

$$\Rightarrow (1-x^2)2\frac{dy}{dx}\frac{d^2y}{dx^2} - 2x\left(\frac{dy}{dx}\right)^2 = m^2.2y\frac{dy}{dx}$$

Dividing each term by $2\frac{dy}{dx}$,

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$$ Proved