

Differentiation

Solution of

Previous Years ISC Class 12 Board questions

2000 to Sem1-2021

1. If $y = \log \sqrt{\frac{1-\cos x}{1+\cos x}}$, find $\frac{dy}{dx}$. (isc 2000)

Solution: Given $y = \log \sqrt{\frac{1-\cos x}{1+\cos x}}$

$$= \log \sqrt{\tan^2 \left(\frac{x}{2}\right)}$$
$$= \log \left(\tan \frac{x}{2}\right)$$
$$\therefore \frac{dy}{dx} = \frac{1}{\tan \left(\frac{x}{2}\right)} \cdot \frac{1}{2} \cdot \sec^2 \left(\frac{x}{2}\right)$$
$$= \frac{1}{2} \cdot \frac{\cos \left(\frac{x}{2}\right)}{\sin \left(\frac{x}{2}\right)} \cdot \frac{1}{\cos^2 \left(\frac{x}{2}\right)}$$
$$= \frac{1}{2 \sin \left(\frac{x}{2}\right) \cos \left(\frac{x}{2}\right)}$$
$$= \frac{1}{\sin x}$$
$$= \text{cosec } x$$

Answer.

2. If $y = (\cos x)^{\cos x}$, find $\frac{dy}{dx}$. (isc 2001)

Solution: Given $y = (\cos x)^{\cos x}$

Taking log both sides

$$\log y = \cos x \log(\cos x)$$

Differentiating each term w. r. t. x

$$\frac{1}{y} \frac{dy}{dx} = \cos x \frac{1}{\cos x} (-\sin x) + (-\sin x) \log(\cos x)$$
$$\Rightarrow \frac{1}{y} \frac{dy}{dx} = -\sin x - \sin x \log(\cos x)$$
$$\Rightarrow \frac{dy}{dx} = -y \sin x \{1 + \log(\cos x)\}$$
$$\Rightarrow \frac{dy}{dx} = -(\cos x)^x \sin x \{\log e + \log(\cos x)\}$$
$$\Rightarrow \frac{dy}{dx} = -(\cos x)^x \sin x \{\log(e \cos x)\}$$

Answer.

3. If $y = e^x \log(\tan 2x)$, find $\frac{dy}{dx}$. (isc 2002)

Solution: Given $y = e^x \log(\tan 2x)$

$$\Rightarrow \frac{dy}{dx} = e^x \frac{1}{\tan 2x} \cdot 2 \sec^2(2x) + e^x \log \tan 2x$$

$$\Rightarrow \frac{dy}{dx} = e^x \{2 \cot 2x \cdot \sec^2(2x) + \log \tan 2x\}$$

Answer.