$$\Rightarrow \quad \frac{dx}{dt} = 3\,a\,sin^2t.\cos t \quad \Big| \quad \Rightarrow \quad \frac{dy}{dt} = -3a\,cos^2t.\sin t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3a\,cos^2t.\sin t}{3\,a\,sin^2t.\cos t}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{\cos t}{\sin t}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\cot t \qquad \text{Answer.}$$

8. *If* $\sin(xy) + \cos(xy) = 1$ *and* $\tan(xy) \neq 1$, *then show that* $\frac{dy}{dx} = -\frac{y}{x}$. *(isc 2005)*

Solution: Given $\sin(xy) + \cos(xy) = 1$

Differentiating $w.r.t.\ x$

$$\Rightarrow \quad \cos(xy)\left(y + x\frac{dy}{dx}\right) - \sin(xy)\left(y + x\frac{dy}{dx}\right) = 0$$

$$\Rightarrow \quad \cos(xy).x\frac{dy}{dx} - \sin(xy).x\frac{dy}{dx} = -\cos(xy).y + \sin(xy).y$$

$$\Rightarrow \quad \{\cos(xy) - \sin(xy)\}\,x\frac{dy}{dx} = y\,\{\sin(xy) - \cos(xy)\}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y\,\{\sin(xy) - \cos(xy)\}}{x\,\{\cos(xy) - \sin(xy)\}}$$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y}{x} \qquad \text{Proved.}$$

9. *If* $x^p\,y^q = (x+y)^{p+q}$, *prove that* $\frac{dy}{dx} = \frac{y}{x}$. *(isc 2005)*

Solution: Given $x^p\,y^q = (x+y)^{p+q}$

Taking log both sides

$$\Rightarrow \quad p\,\log x + q\,\log y = (p+q)\log(x+y)$$

Differentiating each term $w.r.t.\ x$

$$\Rightarrow \quad p.\frac{1}{x} + q\,\frac{1}{y}.\frac{dy}{dx} = (p+q)\frac{1}{x+y}.\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \quad \frac{p}{x} + \frac{q}{y}.\frac{dy}{dx} = \frac{p+q}{x+y} + \frac{p+q}{x+y}.\frac{dy}{dx}$$

$$\Rightarrow \quad \left(\frac{q}{y} - \frac{p+q}{x+y}\right)\frac{dy}{dx} = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\Rightarrow \quad \left(\frac{qx+qy-py-qy}{y(x+y)}\right)\frac{dy}{dx} = \frac{px+qx-px-py}{x\,(x+y)}$$

$$\Rightarrow \quad \frac{qx-py}{y\,(x+y)}.\frac{dy}{dx} = \frac{qx-py}{x\,(x+y)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{(qx-py)\;y\;(x+y)}{x\,(x+y)\,(qx-py)}$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{x} \qquad \text{Proved}$$

10. *If* $y = e^{\sin x^2}$, *find* $\frac{dy}{dx}$. *(isc 2006)*

Solution: Given $y = e^{\sin x^2}$

Taking log both sides

$$\Rightarrow \quad \log y = \sin x^2$$

Differentiating each term $w.r.t.\ x$

$$\Rightarrow \quad \frac{1}{y}.\frac{dy}{dx} = \cos x^2.2x$$

$$\Rightarrow \quad \frac{dy}{dx} = y.2x\,\cos x^2$$