$\Rightarrow \quad \frac{dy}{dx} = 2x.e^{\sin x^2}.\cos x^2$ Answer.

11. $If\ y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}, prove\ that\ (1-x^2)\frac{dy}{dx} - xy = 1$ $(isc\ 2006)$

Solution: Given $y = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$

$\Rightarrow \quad \sqrt{1-x^2}.y = \sin^{-1}x$

$\Rightarrow \quad \sqrt{1-x^2}\ \frac{dy}{dx} + \frac{1}{2\sqrt{1-x^2}}(-2x)y = \frac{1}{\sqrt{1-x^2}}$ (differentiating each term $w.r.t.\ x$)

$\Rightarrow \quad (1-x^2)\frac{dy}{dx} - xy = 1$ Proved.

12. $If\ e^{x+y} = xy, show\ that\ \frac{dy}{dx} = \frac{y(1-x)}{x(1-y)}$ $(isc\ 2007)$

Solution: Given $e^{x+y} = xy$

Taking log both sides

$\Rightarrow \quad x + y = \log x + \log y$

$Differentiating\ each\ term\ w.r.t.\ x$

$\Rightarrow \quad 1 + \frac{dy}{dx} = \frac{1}{x} + \frac{1}{y}.\frac{dy}{dx}$

$\Rightarrow \quad \left(1 - \frac{1}{y}\right)\frac{dy}{dx} = \frac{1}{x} - 1$

$\Rightarrow \quad \left(\frac{y-1}{y}\right)\frac{dy}{dx} = \frac{1-x}{x}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$ Proved

13. $If \quad \sin y = x\ \sin(a+y), show\ that\ \frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$ $(isc\ \ 2007)$

Solution : Given $\sin y = x\ \sin(a+y)$

$\Rightarrow \quad x = \frac{\sin y}{\sin(a+y)}$

Differentiating both sides $w.r.t.\ y$

$\Rightarrow \quad \frac{dx}{dy} = \frac{\sin(a+y)\ \cos y - \cos(a+y)\ \sin y}{sin^2(a+y)}$

$\Rightarrow \quad \frac{dx}{dy} = \frac{\sin(a+y-y)}{sin^2(a+y)}$

$\Rightarrow \quad \frac{dx}{dy} = \frac{\sin a}{sin^2(a+y)}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{sin^2(a+y)}{\sin a}$ Proved

14. $Find\ \frac{dy}{dx}\ \ if\ \ y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$ $(isc\ \ 2008)$

Solution: Given $y = \tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$

$= \tan^{-1}\frac{\sqrt{1+tan^2\theta}-1}{\tan\theta}$

$= \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$Let\ x = \tan\theta$

$\therefore \quad \theta = \tan^{-1}x$