$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2}$$

$$= \frac{1}{2}\tan^{-1}x$$

$$\therefore \frac{dy}{dx} = \frac{1}{2(1+x^2)}$$ Answer

15. $If\ y = \sqrt{\frac{1-\cos x}{1+\cos x}}, find\ \frac{dy}{dx}$ $(isc\ 2008)$

Solution: Given $y = \sqrt{\frac{1-\cos x}{1+\cos x}}$

$$= \sqrt{tan^2\left(\frac{x}{2}\right)}$$

$$= \tan\left(\frac{x}{2}\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}sec^2\left(\frac{x}{2}\right)$$ Answer.

16. Using a suitable substitution, find the derivative of $\tan^{-1}\frac{4\sqrt{x}}{1-4x}$ $(isc\ 2009)$

Solution: Let $y = \tan^{-1}\frac{4\sqrt{x}}{1-4x}$ $Let\ 2\sqrt{x} = \tan\theta$

$$= \tan^{-1}\frac{2.2\sqrt{x}}{1-(2\sqrt{x})^2} \qquad \therefore \theta = \tan^{-1}(2\sqrt{x})$$

$$= \tan^{-1}\left(\frac{2\tan\theta}{1-tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 2\theta)$$

$$= 2\theta$$

$$= 2\tan^{-1}(2\sqrt{x})$$

$$\therefore \frac{dy}{dx} = \frac{2}{1+(2\sqrt{x})^2}.2.\frac{1}{2\sqrt{x}}$$

$$= \frac{2}{\sqrt{x}(1+4x)}$$ Answer.

17. Find the derivative of $\sin x^2$ with respect to x^3. $(isc\ 2009)$

Solution: $Let\ y = \sin x^2$ $and\ z = x^3$