Taking log both sides

$\log y = m\ \cos^{-1} x$

Differentiating $w.r.t.\ x$

$\frac{1}{y}\frac{dy}{dx} = m\left(-\frac{1}{\sqrt{1-x^2}}\right)$

$\Rightarrow \quad \sqrt{1-x^2}\,\frac{dy}{dx} = -m\,y$

squaring both sides

$\Rightarrow \quad (1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2\,y^2$

Differentiating again w.r.t. x

$\Rightarrow \quad (1-x^2)2\,\frac{dy}{dx}\frac{d^2y}{dx^2} + (-2x)\left(\frac{dy}{dx}\right)^2 = m^2\,.2y\,\frac{dy}{dx}$

dividing each term by $2\,\frac{dy}{dx}$

$\Rightarrow \quad (1-x^2)\frac{d^2y}{dx^2} - x\,\frac{dy}{dx} = m^2\,y$ Proved.

24. *If* $\log y = \tan^{-1} x$ *, prove that* $(1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$ (*isc* 2016)

Solution: Given $\log y = \tan^{-1} x$

Differentiating both sides $w.r.t.\ x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{1+x^2}$

$\Rightarrow \quad (1+x^2)\frac{dy}{dx} = y$

Differentiating again w.r.t x

$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + 2x\,\frac{dy}{dx} = \frac{dy}{dx}$

$\Rightarrow \quad (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$ Proved

25. *If* $y = \cos(\sin x)$*, show that* $\frac{d^2y}{dx^2} + \tan x\,\frac{dy}{dx} + y\,cos^2x = 0$ (*isc* 2017)

Solution: Given $y = \cos(\sin x)$

Differentiating both sides $w.r.t.\ x$

$\Rightarrow \quad \frac{dy}{dx} = -\sin(\sin x)\,.\cos x$

again differentiating both sides w.r.t. x

$\Rightarrow \quad \frac{d^2y}{dx^2} = -\cos(\sin x)\,.\cos x\,.\cos x - \sin(\sin x)\,.(-\sin x)$

$\Rightarrow \quad \frac{d^2y}{dx^2} = -\cos(\sin x)\,.cos^2x + \sin(\sin x)\,.\sin x$

Now $L.H.S = \frac{d^2y}{dx^2} + \tan x\,\frac{dy}{dx} + y\,cos^2x$

$= -\cos(\sin x)\,.cos^2x + \sin(\sin x)\,.\sin x + \frac{\sin x}{\cos x}.\{-\sin(\sin x)\,.\cos x\} + \cos(\sin x)\,.cos^2x$

$= -\cos(\sin x)\,.cos^2x + \sin(\sin x)\,.\sin x - \sin x\,.\sin(\sin x) + \cos(\sin x)\,.cos^2x$

$= 0$ Proved