26. *If* $x = \tan\left(\frac{1}{a}\log y\right)$ *, prove that* $(1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$ (*isc* 2018)

Solution: Given $x = \tan\left(\frac{1}{a}\log y\right)$

$\Rightarrow \tan^{-1}x = \frac{1}{a}\log y$

$\Rightarrow \log y = a\tan^{-1}x$

Differentiating both sides $w.r.t.\ x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = a\frac{1}{1+x^2}$

$\Rightarrow (1+x^2)\frac{dy}{dx} = a\,y$

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = a\frac{dy}{dx}$ (Differentiating each term $w.r.t.x$)

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-a)\frac{dy}{dx} = 0$ Proved

27. *If* $y = e^{\sin^{-1}x}$ *and* $z = e^{-\cos^{-1}x}$*, prove that* $\frac{dy}{dz} = e^{\pi/2}$ (*isc* 2019)

Solution: $y = e^{\sin^{-1}x}$ and $z = e^{-\cos^{-1}x}$

$\Rightarrow \frac{dy}{dx} = e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}$ and $\frac{dz}{dx} = e^{-\cos^{-1}x}.\left(-\frac{1}{-\sqrt{1-x^2}}\right)$

$\therefore \frac{dy}{dz} = \frac{\frac{dy}{dx}}{\frac{dz}{dx}} = \frac{e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}}{e^{-\cos^{-1}x}.\frac{1}{\sqrt{1-x^2}}}$

$= \frac{e^{\sin^{-1}x}}{e^{-\cos^{-1}x}}$

$= e^{\sin^{-1}x+\cos^{-1}x}$

$= e^{\pi/2}$ Proved

28. *Find* $\frac{dy}{dx}$ *if* $x^3+y^3 = 3axy$ (*isc* 2020)

Solution: Given $x^3+y^3 = 3axy$

Differentiating both sides $w.r.t.x,$

$3x^2+3y^2\frac{dy}{dx} = 3a\left(x\frac{dy}{dx}+y\right)$

$\Rightarrow x^2+y^2\frac{dy}{dx} = ax\frac{dy}{dx}+ay$

$\Rightarrow (y^2-ax)\frac{dy}{dx} = ay-x^2$

$\Rightarrow \frac{dy}{dx} = \frac{ay-x^2}{y^2-ax}$ Answer

29. *If* $y = e^{m\sin^{-1}x}$*, prove that* $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2\,y$ (*isc* 2020)

Solution: Given $y = e^{m\sin^{-1}x}$

Taking log on both sides

$\Rightarrow \log y = m\sin^{-1}x$

Differentiating each term $w.r.t.x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{m}{\sqrt{1-x^2}}$ $\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = m\,y$

$\Rightarrow (1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2\,y^2$ (squaring both sides)