5. $x = \cos\theta - \cos 2\theta\,, \quad y = \sin\theta - \sin 2\theta$

Solution: Given that $x = \cos\theta - \cos 2\theta$ and $y = \sin\theta - \sin 2\theta$

$\therefore \quad \frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$ and $\frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$

Hence, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta} = \mathbf{\frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}}$

6. $x = a(\theta - \sin\theta), \quad y = a(1 + \cos\theta)$

Solution: Given that $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$

$\therefore \quad \frac{dx}{d\theta} = a(1 - \cos\theta)$ and $\frac{dy}{d\theta} = -a\sin\theta$

Hence, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-a\sin\theta}{a(1-\cos\theta)} = \frac{-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\left(\frac{\theta}{2}\right)} = \mathbf{-\cot\frac{\theta}{2}}$

7. $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}\,, \quad y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

Solution:

$$x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$$

$$\therefore \quad \frac{dx}{dt} = \frac{\sqrt{\cos 2t}\,.\,\frac{d}{dt}(\sin^3 t) - \sin^3 t\,.\,\frac{d}{dt}(\sqrt{\cos 2t})}{\left(\sqrt{\cos 2t}\right)^2}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{\sqrt{\cos 2t}\,.\,3\sin^2 t\,.\,\cos t - \sin^3 t\,.\,\frac{1}{2\sqrt{\cos 2t}}.(-2\sin 2t)}{\cos 2t}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{\frac{3\cos 2t\,.\,\sin^2 t\,.\,\cos t + \sin^3 t\,.\,\sin 2t}{\sqrt{\cos 2t}}}{\cos 2t}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{3\left(1 - 2\sin^2 t\right)\sin^2 t\,.\,\cos t + \sin^3 t\,.\,2\sin t\,.\,\cos t}{\cos 2t\,.\,\sqrt{\cos 2t}}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{3\sin^2 t\,.\,\cos t - 6\sin^4 t\,.\,\cos t + 2\sin^4 t\,.\,\cos t}{\cos 2t\,.\,\sqrt{\cos 2t}}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{3\sin^2 t\,.\,\cos t - 4\sin^4 t\,.\,\cos t}{\cos 2t\,.\,\sqrt{\cos 2t}}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{\sin t\,.\,\cos t\,(3\sin t - 4\sin^3 t)}{\cos 2t\,.\,\sqrt{\cos 2t}}$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{\sin t\,.\,\cos t\,.\,\sin 3t}{\cos 2t\,.\,\sqrt{\cos 2t}} \qquad \ldots\ldots\ldots\ldots \quad (1)$$

Now $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{\sqrt{\cos 2t}\,.\,\frac{d}{dt}(\cos^3 t) - \cos^3 t\,.\,\frac{d}{dt}(\sqrt{\cos 2t})}{\left(\sqrt{\cos 2t}\right)^2}$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{\sqrt{\cos 2t}\,.\,3\cos^2 t\,.(-\sin t) - \cos^3 t\,.\,\frac{1}{2\sqrt{\cos 2t}}.\,(-2\sin 2t)}{\cos 2t}$$

$$\Rightarrow \quad \frac{dy}{dt} = \frac{\frac{-3\cos 2t\,.\,\cos^2 t\,.\,\sin t + \cos^3 t\,.\,2\sin t\,.\,\cos t}{\sqrt{\cos 2t}}}{\cos 2t}$$

Tapati's Classes

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