$$\Rightarrow \frac{dy}{dt} = \frac{-3\left(2\cos^2 t - 1\right)\cos^2 t \,.\sin t + 2\cos^4 t \,.\sin t}{\cos 2t \,.\sqrt{\cos 2t}}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-6\cos^4 t \,.\sin t + 3\cos^2 t \,.\sin t + 2\cos^4 t \,.\sin t}{\cos 2t \,.\sqrt{\cos 2t}}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-4\cos^4 t \,.\sin t + 3\cos^2 t \,.\sin t}{\cos 2t \,.\sqrt{\cos 2t}}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-\sin t \,.\cos t\left(4\cos^3 t - 3\cos t\right)}{\cos 2t \,.\sqrt{\cos 2t}}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-\sin t \,.\cos t \,.\cos 3t}{\cos 2t \,.\sqrt{\cos 2t}} \qquad \ldots\ldots\ldots\ldots (2)$$

$\therefore$ From (1) and (2)

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-\sin t \,.\cos t \,.\cos 3t}{\cos 2t \,.\sqrt{\cos 2t}}}{\frac{\sin t \,.\cos t \,.\sin 3t}{\cos 2t \,.\sqrt{\cos 2t}}} = -\frac{\cos 3t}{\sin 3t} = \mathbf{-\cot 3t}$$

8. $x = a\left(\cos t + \log\tan\frac{t}{2}\right), \quad y = a\sin t$

Solution:

Given $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$

Differentiating $w.r.t.\ t$:

$$\frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}} \,.\frac{1}{2} \,.sec^2\frac{t}{2}\right)$$

$$= a\left(-\sin t + \frac{\cos\frac{t}{2}}{2\sin\frac{t}{2}\cos^2\frac{t}{2}}\right)$$

$$= a\left(-\sin t + \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right)$$

$$= a\left(-\sin t + \frac{1}{\sin t}\right)$$

$$= a\left(\frac{-\sin^2 t + 1}{\sin t}\right)$$

$$= a\frac{\cos^2 t}{\sin t}$$

Now $y = a\sin t$

Differentiating $w.r.t.\ t$:

$$\frac{dy}{dt} = a\cos t$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\frac{\cos^2 t}{\sin t}} = \frac{\sin t}{\cos t} = \mathbf{\tan t}$$

9. $x = a\sec\theta, \quad y = b\tan\theta$

Solution:

Given that $x = a\sec\theta$ and $y = b\tan\theta$

$\therefore \frac{dx}{d\theta} = a\sec\theta\tan\theta$ and $\frac{dy}{d\theta} = b\,sec^2\theta$

Hence, $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{b\,sec^2\theta}{a\sec\theta\tan\theta} = \frac{b\,\sec\theta}{a\,\tan\theta} = \frac{b\times\cos\theta}{a\,\cos\theta\times\sin\theta} = \mathbf{\frac{b}{a}\,cosec\,\theta}$