Solutions of

Linear Regression

ISC Previous Years Board Questions

2005 to 2013

1. There are two series of index numbers: P for price index and S for stock of a commodity. The mean and the standard deviation of P are 100 and 8 and of S are 103 and 4 respectively. The correlation coefficient between the two series is 0.4. With these data, obtain the regression lines of P on S and S on P.

(ISC 2005)

Solution: Given

Commodity	Price P (x)	Stock S (y)
Mean	$\bar{x} = 100$	$\bar{y} = 103$
Standard deviation	$\sigma_x = 8$	$\sigma_y = 4$
Correlation coefficient	$r = 0.4$	

Regression coefficient of S on P is $b_{yx} = r\,\frac{\sigma_y}{\sigma_x} = 0.4 \times \frac{4}{8} = 0.2$

Regression coefficient of P on S is $b_{xy} = r\,\frac{\sigma_x}{\sigma_y} = 0.4 \times \frac{8}{4} = 0.8$

Hence the regression line of P on S is (i.e., regression equation of x on y)

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$\Rightarrow \quad x - 100 = 0.8\,(y - 103)$$

$$\Rightarrow \quad x - 100 = 0.8\,y - 82.4$$

$$\Rightarrow \quad x = 0.8\,y + 17.6$$

The regression line of S on P is (i.e., regression equation of y on x)

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\Rightarrow \quad y - 103 = 0.2\,(x - 100)$$

$$\Rightarrow \quad y = 0.2\,x - 20 + 103$$

$$\Rightarrow \quad y = 0.2\,x + 83$$

2. Out of the two regression lines, find the line of regression of y on x:

$3x + 12y = 9,\ \ 9x + 3y = 46$ (ISC 2005)

Solution: Let equation $3x + 12y = 9$ is the regression equation of y on x and equation $9x + 3y = 46$ is the regression equation of x on y.

$$3x + 12y = 9$$

$$\Rightarrow \quad 12\,y = -3x + 9$$

$$\Rightarrow \quad y = -\frac{3}{12}\,x + \frac{9}{12}$$

$$\Rightarrow \quad y = -\frac{1}{4}\,x + \frac{3}{4}$$

$$\therefore \quad b_{yx} = -\frac{1}{4}$$

$$9x + 3y = 46$$

$$\Rightarrow \quad 9x = -3y + 46$$

$$\Rightarrow x = -\frac{3}{9}\,y + \frac{46}{9}$$

$$\Rightarrow x = -\frac{1}{3}\,y + \frac{46}{9}$$

$$\therefore \quad b_{xy} = -\frac{1}{3}$$

$$\therefore\ r^2 = -\frac{1}{4} \times -\frac{1}{3} = \frac{1}{12} \qquad \left[0 < \frac{1}{12} < 1\right]$$

We know $\quad 0 \le r^2 \le 1$