6. If the two regression lines of a bivariate distribution are $4x - 5y + 33 = 0$ and $20x - 9y - 107 = 0$,

(i) Calculate, $\bar{x}$ and $\bar{y}$, the arithmetic means of x and y respectively.

(ii) Estimate the value of x when $y = 7$.

(iii) Find the variance of y when $\sigma_x = 3$. (ISC 2009)

Solution:

(i) $4x - 5y + 33 = 0$ ------------- (1)

$20x - 9y - 107 = 0$ --------- (2)

Multiplying equation (1) by 5,

$$20x - 25y + 165 = 0$$
$$20x - 9y - 107 = 0$$
$$- \quad + \quad +$$
$$-16y + 272 = 0 \quad \Rightarrow y = 17 \text{ i.e., } \bar{y} = 17$$

Putting $y = 17$ in (1), $x = 13$, i.e., $\bar{x} = 13$

$\therefore \quad \bar{x} = 13, \ \bar{y} = 17$

(ii) Let equation (1) is the regression equation of y on x and equation (2) is the regression equation of x on y.

$4x - 5y + 33 = 0$

$\Rightarrow \quad 5y = 4x + 33$

$\Rightarrow \quad y = \frac{4}{5}x + \frac{33}{5}$

$\therefore \quad b_{yx} = \frac{4}{5}$

$20x - 9y - 107 = 0$

$\Rightarrow \quad 20x = 9y + 107$

$\Rightarrow \quad x = \frac{9}{20}y + \frac{107}{20}$

$\therefore \quad b_{xy} = \frac{9}{20}$

$\therefore \quad r^2 = b_{yx} \times b_{xy} = \frac{4}{5} \times \frac{9}{20} = \frac{9}{25}$ $\left[0 < \frac{9}{25} < 1\right]$ (Assumption correct)

$\therefore r = \sqrt{\frac{9}{25}} = \frac{3}{5}$ (r is positive as both b_{yx}, b_{xy} are positive)

$\therefore$ the regression equation of x on y is $20x - 9y - 107 = 0$

$\Rightarrow \quad 20x = 9y + 107$

$\Rightarrow \quad x = \frac{9}{20}y + \frac{107}{20}$

When $y = 7, \ x = \frac{9}{20} \times 7 + \frac{107}{20} = \frac{170}{20} = 8{\cdot}5$

(iii) $b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} \quad \Rightarrow \frac{4}{5} = \frac{3}{5} \times \frac{\sigma_y}{3} \quad \Rightarrow \sigma_y = 4$

$\therefore$ the variance of $y = \sigma_y^2 = 16$

7. If the regression equation of x on y is given by $mx - y + 10 = 0$ and the equation of y on x is given by $-2x + 5y = 14$, determine the value of $'m'$ if the coefficient of correlation between x & y is $\frac{1}{\sqrt{10}}$. (ISC 2010)

Solution: The regression equation of x on y is given by $mx - y + 10 = 0$

$\Rightarrow \quad mx = y - 10$

$\Rightarrow \quad x = \frac{1}{m}y - \frac{10}{m}$

$\therefore \quad b_{xy} = \frac{1}{m}$

The regression equation of y on x is given by $-2x + 5y = 14$

$\Rightarrow \quad 5y = 2x + 14$

$\Rightarrow \quad y = \frac{2}{5}x + \frac{14}{5}$

$\therefore \quad b_{yx} = \frac{2}{5}$