Given $r = \frac{1}{\sqrt{10}}$. We know $r^2 = b_{yx} \times b_{xy}$

$$\therefore \quad \frac{1}{10} = \frac{1}{m} \times \frac{2}{5}$$

$$\therefore m = 4$$

8. Find the equations of the two lines of regression for the following observations:

(3, 6), (4, 5), (5, 4), (6, 3), (7, 2)

Find an estimate of y for $x = 2.5$. (ISC 2010)

Solution:

x	y	x^2	y^2	xy
3	6	9	36	18
4	5	16	25	20
5	4	25	16	20
6	3	36	9	18
7	2	49	4	14
$\sum x = 25$	$\sum y = 20$	$\sum x^2 = 135$	$\sum y^2 = 90$	$\sum xy = 90$

$n = 5$, $\bar{x} = \frac{\sum x}{n} = \frac{25}{5} = 5$, $\bar{y} = \frac{\sum y}{n} = \frac{20}{5} = 4$

Regression coefficient of y on x $= b_{yx} = \frac{\sum x y - \frac{\sum x \cdot \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{90 - \frac{25 \times 20}{5}}{135 - \frac{25^2}{5}} = \frac{-10}{10}$

$= -1$

Regression coefficient of x on y $= b_{xy} = \frac{\sum x y - \frac{\sum x \cdot \sum y}{n}}{\sum y^2 - \frac{(\sum y)^2}{n}} = \frac{90 - \frac{25 \times 20}{5}}{90 - \frac{20^2}{5}} = \frac{-10}{10}$

$= -1$

The regression line of y on x is $y - \bar{y} = b_{yx}(x - \bar{x})$

$\Rightarrow y - 4 = -1(x - 5)$

$\Rightarrow x + y = 9$

The regression line of of x on y is given by $(x - \bar{x}) = b_{xy}(y - \bar{y})$

$\Rightarrow x - 5 = -1(y - 4)$

$\Rightarrow x + y - 9 = 0$

We have to estimate y when $x = 2.5$.

The regression line of y on x is $x + y = 9$

$\Rightarrow y = -x + 9$

$\Rightarrow y = -2.5 + 9 = 6.5$

9. Two regression lines are represented by $2x + 3y - 10 = 0$ and $4x + y - 5 = 0$. Find the line of regression of y on x. (ISC 2011)

Solution: Let $2x + 3y - 10 = 0$ is the regression line of y on x and $4x + y - 5 = 0$ is the regression line of x on y.

$2x + 3y - 10 = 0$

$\Rightarrow 3y = -2x + 10$

$\Rightarrow y = -\frac{2}{3}x + \frac{10}{3}$

$\therefore b_{yx} = -\frac{2}{3}$

$4x + y - 5 = 0$

$\Rightarrow 4x = -y + 5$

$\Rightarrow x = -\frac{1}{4}y + \frac{5}{4}$

$\therefore b_{xy} = -\frac{1}{4}$