$$r^2 = b_{yx} \times b_{xy} = \left(-\frac{2}{3}\right)\left(-\frac{1}{4}\right) = \frac{1}{6}, \qquad 0 < \frac{1}{6} < 1$$

We know $0 \le r^2 \le 1$, so our assumption is correct.

Hence $2x + 3y - 10 = 0$ is the regression line of y on x.

10. The following observations are given:

(1, 4), (2, 8), (3, 2), (4, 12), (5, 10), (6, 14), (7, 16), (8, 6), (9, 18)

Estimate the value of y when the value of x is 10 add also estimate the value of x when the value of y is 5. (ISC 2011)

Solution:

$$\bar{x} = \frac{\sum x}{n} = \frac{45}{9} = 5 \text{ and } \bar{y} = \frac{\sum y}{n} = \frac{90}{9} = 10$$

x	y	$x - \bar{x} = x - 5$	$y - \bar{y} = y - 10$	$(x - x)^2 = (x-5)^2$	$(y - \bar{y})^2 = (y-10)^2$	$(x - \bar{x})(y - \bar{y})$
1	4	− 4	− 6	16	36	24
2	8	− 3	− 2	9	4	6
3	2	− 2	− 8	4	64	16
4	12	− 1	2	1	4	− 2
5	10	0	0	0	0	0
6	14	1	4	1	16	4
7	16	2	6	4	36	12
8	6	3	− 4	9	16	− 12
9	18	4	8	16	64	32
$\sum x = 45$	$\sum y = 90$			$\sum(x-\bar{x})^2 = 60$	$\sum(y-\bar{y})^2 = 240$	$\sum(x-\bar{x})(y-\bar{y}) = 80$

$$b_{yx} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2} = \frac{80}{60} = \frac{4}{3}$$

$$b_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(y-\bar{y})^2} = \frac{80}{240} = \frac{1}{3}$$

The regression equation of y on x is

$$y - \bar{y} = b_{yx}(x - \bar{x})$$

$$\Rightarrow y - 10 = \frac{4}{3}(x - 5)$$

$$\Rightarrow y = \frac{4}{3}x - \frac{20}{3} + 10$$

$$\Rightarrow y = \frac{4}{3}x + \frac{10}{3}$$

When $x = 10,\ y = \frac{4}{3} \times 10 + \frac{10}{3} = \frac{50}{3}$

The regression equation of x on y is

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$\Rightarrow x - 5 = \frac{1}{3}(y - 10)$$

$$\Rightarrow x = \frac{1}{3}y + 5 - \frac{10}{3}$$

$$\Rightarrow x = \frac{1}{3}y + \frac{5}{3}$$

When $y = 5,\ x = \frac{5}{3} + \frac{5}{3} = \frac{10}{3}$