Form	Form of the partial fraction
$\dfrac{px+q}{(x-a)(x-b)}$	$\dfrac{A}{x-a}+\dfrac{B}{x-b}$

Let $\dfrac{2x+1}{(x-1)(x+1)} = \dfrac{A}{x-1} + \dfrac{B}{x+1}$

$\therefore \dfrac{2x+1}{(x-1)(x+1)} = \dfrac{A(x+1)+B(x-1)}{(x-1)(x+1)}$

$\therefore 2x + 1 = A(x+1) + B(x-1)$

Put $x = 1,\ 2+1 = A(1+1) \Rightarrow A = \frac{3}{2}$

Put $x = -1,\ -2+1 = B(-1-1) \Rightarrow B = \frac{1}{2}$

$\therefore \dfrac{2x+1}{(x-1)(x+1)} = x + \dfrac{\frac{3}{2}}{x-1} + \dfrac{\frac{1}{2}}{x+1}$

$$\therefore \int \frac{2x+1}{(x-1)(x+1)}\,dx = \int x\,dx + \frac{3}{2}\int \frac{1}{x-1}\,dx + \frac{1}{2}\int \frac{1}{x+1}\,dx$$

$$= \frac{x^2}{2} + \frac{3}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$$

13. $\dfrac{2}{(1-x)(1+x^2)}$

Solution:

Form	Form of the partial fraction
$\dfrac{px^2+qx+r}{(x-a)(x^2+bx+c)}$	$\dfrac{A}{x-a}+\dfrac{Bx+C}{x^2+bx+c}$
Where (x^2+bx+c) cannot be factorised further	

Let $\dfrac{2}{(1-x)(1+x^2)} = \dfrac{A}{1-x} + \dfrac{Bx+C}{1+x^2}$

$\Rightarrow \dfrac{2}{(1-x)(1+x^2)} = \dfrac{A(1+x^2)+(Bx+C)(1-x)}{(1-x)(1+x^2)}$

$\Rightarrow 2 = A(1+x^2) + (Bx+C)(1-x)$

Put $x = 1,\ 2 = A(1+1) \Rightarrow A = 1$

Equating coefficients of x^2 on both sides, $0 = A - B \Rightarrow B = A = 1$

Equating coefficients of x^0 on both sides,

$$2 = A + C \Rightarrow C = 2 - A = 2 - 1 = 1$$

$\therefore \dfrac{2}{(1-x)(1+x^2)} = \dfrac{1}{1-x} + \dfrac{x+1}{1+x^2}$

$$\therefore \int \frac{2}{(1-x)(1+x^2)}\,dx = \int \frac{1}{1-x}\,dx + \int \frac{x+1}{1+x^2}\,dx$$

$$= \int \frac{1}{1-x}\,dx + \int \frac{x}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx$$

$$= \int \frac{1}{1-x}\,dx + \frac{1}{2}\int \frac{2x}{1+x^2}\,dx + \int \frac{1}{1+x^2}\,dx$$

$$= -\log|1-x| + \frac{1}{2}\log(1+x^2) + \tan^{-1}x + C$$

14. $\dfrac{3x-1}{(x+2)^2}$