

5. Evaluate the following determinant without expanding.

(ISC 2023)

$$\begin{vmatrix} 5 & 5 & 5 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

Solution:

$$\begin{vmatrix} 5 & 5 & 5 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix}$$

$$= 5 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{vmatrix} \quad (\text{Taking common from } R_1)$$

$$= 5 \begin{vmatrix} 1 & 1 & 1 \\ a+b+c & b+c+a & c+a+b \\ b+c & c+a & a+b \end{vmatrix}$$

$(R_2 \rightarrow R_2 + R_3)$

$$= 5(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ b+c & c+a & a+b \end{vmatrix}$$

$(\text{Taking common from } R_2)$

$$= 5(a+b+c) \times 0 = 0$$

$(R_1 \text{ and } R_2 \text{ are identical, so value of determinant is zero})$