Solutions of

Maxima and Minima

ISC Class 12 Previous Years Board Questions

2020 to 2023

1. Show that the radius of a closed right circular cylinder of given surface area and maximum volume is equal to half of its height. (ISC 2020)

Solution: Let r be the radius of the base and h be the height of the closed circular cylinder.

Let S be the surface area and V be the volume of the cylinder.

$\therefore \quad S = 2\,\pi\,r^2 + 2\,\pi\,r\,h$ ------ (i)

⇨ $2\,\pi\,r\,h = S - 2\,\pi\,r^2$

⇨ $h = \frac{S}{2\,\pi\,r} - r$

and $V = \pi\,r^2\,h$

⇨ $V = \pi\,r^2\left(\frac{S}{2\,\pi\,r} - r\right)$

⇨ $V = \frac{S}{2}\,r - \pi\,r^3$

Differentiating $w.r.t.\ r$,

$\frac{dV}{dr} = \frac{S}{2} - 3\,\pi\,r^2$

Again differentiating $w.r.t.\ r$,

$\frac{d^2V}{dr^2} = 0 - 6\,\pi\,r = -6\,\pi\,r < 0$

$\therefore$ For maximum volume

$\frac{dV}{dr} = 0$

⇨ $\frac{S}{2} - 3\,\pi\,r^2 = 0$

⇨ $\frac{S}{2} = 3\,\pi\,r^2$ or $S = 6\,\pi\,r^2$

⇨ $2\,\pi\,r^2 + 2\,\pi\,r\,h = 6\,\pi\,r^2$ [from (i)]

⇨ $2\,\pi\,r\,h = 4\,\pi\,r^2$

⇨ $h = 2\,r$ or $r = \frac{h}{2}$

Radius = $\frac{1}{2}$ × height Proved