2. Prove that the area of a right-angled triangle of given hypotenuse is maximum when the triangle is isosceles. (ISC 2020)

Solution: Let a, b and l be the three sides of the right angled triangle ABC.

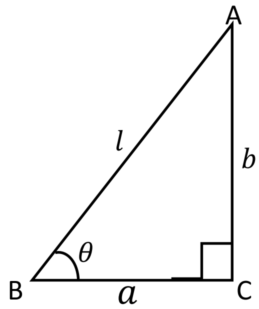
From figure, $a = l\ \cos\theta$ and $b = l\ \sin\theta$

Area of the triangle $A = \frac{1}{2}\ a\ b$

$$= \frac{1}{2}\ l\ \cos\theta\ .\ l\sin\theta$$

$$= \frac{1}{4}\ l^2 .2\ \cos\theta\sin\theta$$

$$= \frac{1}{4}\ l^2\ \sin 2\theta$$



Differentiating $w.r.t.\ \theta$, $\frac{dA}{d\theta} = \frac{1}{2}\ l^2\ \cos 2\theta$

And $\frac{d^2A}{d\theta^2} = -\ l^2 \sin 2\theta$

As θ is acute, $\sin 2\theta > 0$, so $\frac{d^2A}{d\theta^2} < 0$

$\therefore$ For maximum area of the triangle $\frac{dA}{d\theta} = 0$

$$\Rightarrow \quad \frac{1}{2} l^2 \cos 2\theta = 0$$

$$\Rightarrow \quad \cos 2\theta = 0\ , \quad l \neq 0$$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \theta = \frac{\pi}{4}$$

$$\therefore a = l\ \cos\left(\frac{\pi}{4}\right) = \frac{l}{\sqrt{2}} \quad and \quad b = l\ \sin\left(\frac{\pi}{4}\right) = \frac{l}{\sqrt{2}}$$

$$\therefore \quad a = b = \frac{l}{\sqrt{2}}$$

Therefore, the area of a right-angled triangle of a given hypotenuse is maximum when the triangle is isosceles. Proved.

3. A person has manufactured a water tank in the shape of a closed right circular cylinder. The volume of the cylinder is $\frac{539}{2}$ cubic units. If the height and radius of the cylinder be h and r, then: (ISC Sem1-2021)

(i) The height h in terms of radius r and the given volume will be:

(a) $h = \frac{539}{\pi\, r^2}$

(b) $h = \frac{539}{2\,\pi\, r^2}$

(c) $h = \frac{539}{2\,\pi\, r}$

(d) $h = \frac{539}{\pi\, r}$

Answer: ______(b)____________

Solution: $\pi\, r^2\, h = \frac{539}{2} \Rightarrow h = \frac{539}{2\,\pi\, r^2}$